Double-slit X-ray dynamical diffraction in curved crystals

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The theoretical investigation of double-slit X-ray dynamical diffraction in curved crystals shows that Young’s interference fringes are formed. An expression for the period of the fringes has been established which is polarization sensitive. The position of the fringes in the cross section of the beam depends on the deviation from the Bragg exact orientation for a perfect crystal, on the curvature radius and on the thickness of the crystal. This type of diffraction can be used for determination of the curvature radius by measuring the shift of the fringes from the centre of the beam.

1. Introduction

As is well known, there are interferometers of amplitude division and of wavefront division types (Born & Wolf, 2002). Some interferometers of wavefront division type use two narrow slits for forming the two interfering beams. The obtained interference pattern is called Young’s fringes. Another method of forming interference fringes using wavefront division is to make two virtual point sources by two separated half-lenses. For X-rays both types of interferometers have been suggested and are nowadays in use. These interferometers use either the classical two-slit setup (pinholes) or two virtual sources (Leitenberger et al., 2001, 2004; Yamazaki & Ishikawa, 2003; Leitenberger & Pietsch, 2007; Tsuji et al., 2009; Isakovic et al., 2010). Compound refractive lens (CRL) arrays for forming two or more interfering sources have also been used (Snigirev et al., 2009, 2014; Zverev et al., 2020). One of the difficulties with X-rays is the necessity for sufficiently large distances for overlapping the interfering beams. On the other hand, it is well known (Authier, 2001; Pinsker, 1982) that, under two-wave X-ray dynamical diffraction conditions in crystals, an incident beam with a few arcseconds of divergence fills a region with an angular range \( \Delta \theta \), where \( \theta \) is the Bragg angle. Thus a crystal thickness of the order of \( \text{mm} \) is sufficient to form a large area of interference for beams.

Dynamical diffraction effects using a receiving slit have been theoretically and experimentally studied (Kato & Lang, 1959; Homma et al., 1966; Kato, 1961a,b; Authier et al., 1968; Hart & Milne, 1968; Kohra & Kikuta, 1968; Slobodetskii et al., 1968). In the work of Balyan (2010) an X-ray dynamical diffraction double-slit interferometer has been suggested and theoretically investigated. An incident X-ray wave passing through a double-slit system diffracts in a crystal, forming an angle with diffracting planes close to the Bragg angle. In this interferometer the corresponding Young’s fringes are formed and an analytical formula has been obtained for the period of the fringes. The fringe period, in contrast to optics, depends on...
the polarization state of the radiation. The interferometer can be used for measurements of refractive index. In addition, the defects and deformations of the crystal, in which the fringes are formed, can be investigated using such a type of interferometer. Kohn & Smirnova (2021, 2022) have suggested and theoretically investigated X-ray dynamical diffraction wavefront division interferometers. In these interferometers two coherent focused points on the surface of a crystal, formed by means of a focusing first crystal or a CRL, are used. The waves emerging from these sources then diffract in a crystal and as a sequence dynamic diffraction Young’s fringes are formed. The period is the same as in the case of two slits. The peculiarities and application of such fringes are studied.

X-ray double-slit dynamical diffraction in comparison with classical double-slit diffraction has an important advantage. The elastic deformations and imperfections of the crystal directly influence the intensity distribution, obtained as the result of the interference between waves emerging from the slits. Thus, double-slit dynamical diffraction provides an opportunity to investigate dynamical diffraction effects, arising from various defects and deformations. In comparison with the dynamical diffraction contrast of defects and deformations, obtained with a slit, the double-slit experiment offers a more sensitive tool for investigations. In addition, more effects can be established and used with contrast from two slits than the contrast obtained with one slit. The double-slit dynamical diffraction contrast can be simulated for known deformation fields and defects, such as elastically bent crystals, dislocations and so on. The obtained distributions of the intensity can be compared with the corresponding contrasts obtained in the experiment. Also it must be pointed out that, for imperfections located at the region near one of the slits, the interference pattern can be interpreted as a hologram. The numerical reconstruction of the wave, passing through the deformed region, in principle gives an opportunity to reconstruct the deformation field. This means that Takagi’s equations can be used for solving the inverse problem.

In this work, a double-slit X-ray transmission (Laue) symmetrical case dynamical diffraction interferometer for the curved crystal case is theoretically investigated. In such a system interference fringes are formed as well. In the symmetrical case the period is the same as in a perfect crystal. The position of the fringes in the beam depends on the thickness, on the crystal curvature radius as well as on the deviation from the Bragg exact orientation. By measuring the shift of the fringes from the centre of the beam, the curvature radius can be determined for known thickness and deviation from the Bragg orientation.

2. The scheme and the theoretical background

Under two-wave dynamical diffraction conditions the electric wavefield inside a crystal is presented as $E = \left[ E_0 \exp(iK_0r) + E_h \exp(iK_hr) \right] \exp[i\chi_0z/(2\cos \theta)]$ (Takagi, 1969; Authier, 2001; Pinsker, 1982), where $E_0, E_h$ are slowly varying amplitudes, $K_0$ and $K_h = K_0 + h$ are the wavevectors of the transmitted and diffracted waves, respectively, $z$ is the coordinate along the reflecting atomic planes, $\chi_0$ is the zero-order Fourier component of the crystal susceptibility. We choose these wavevectors to satisfy the exact Bragg condition

$K_0^2 = K_h^2 = (2\pi/\lambda)^2 = k^2$, where $\lambda$ is the wavelength. Then the amplitudes for $\sigma$-polarized waves satisfy the following Takagi equations (Takagi, 1969):

$$\frac{2i}{k} \frac{\partial E_0}{\partial s_0} + \chi_h E_h \exp(\imath hu) = 0,$$

$$\frac{2i}{k} \frac{\partial E_h}{\partial s_h} + \chi_h E_0 \exp(-\imath hu) = 0. \quad (1)$$

Here, $\mathbf{u}$ is the displacement of atoms from their equilibrium positions in a perfect crystal, $\mathbf{h}$ is the diffraction vector and in the symmetrical Laue case is antiparallel to the Ox axis (Fig. 1). $\chi_{h,k}$ are the crystal Fourier coefficients of susceptibility, $s_0$ and $s_h$ are coordinates along the unit vectors $s_0 = K_0/k$ and $s_h = K_h/k$. For $\pi$-polarized waves $\chi_{h,k}$ must be replaced by $\chi_{h,k} \cos 2\theta$. The crystal is curved around the Oy axis, which is perpendicular to the diffraction plane Oxz. A beam passing through two slits falls on the crystal close to the Bragg exact orientation (Fig. 1).

The relations between $s_0, s_h$ and rectangular coordinates are given by

$$s_0 = \frac{1}{2} \left( \frac{z}{\cos \theta} + \frac{x}{\sin \theta} \right), \quad s_h = \frac{1}{2} \left( \frac{z}{\cos \theta} - \frac{x}{\sin \theta} \right). \quad (2)$$

For homogeneous media the $u_z$ component of the displacement can be written in a simple form (Lekhnitskii, 1981),

$$u_z = \frac{x(z - T/2)}{R}, \quad (3)$$

and thus

$$h_z u_z = f_1(s_0) + f_2(s_h), \quad (3a)$$

where $f_1(s_0) = -k \sin \theta (s_0 \sin 2\theta - T_0 \sin \theta)/R$ and $f_2(s_h) = -k \sin \theta (-s_0 \sin 2\theta + T_h \sin \theta)/R$. Here $T$ is the crystal thickness and $R$ is the curvature radius. Thus the displacement can

![Figure 1](image-url)
be represented as a sum of functions on \( s_0 \) and \( s_a \). It is well known that in this case the equations (1) can be reduced to the ideal crystal case (Pinsker, 1982). Let us in (1) pass to the functions

\[
E_0' = E_0 \exp[-if_z(s_h)], \quad E_h' = E_h \exp[if_i(s_0)].
\]  
(4)

Then the equations (1) can be written in the form of equations in a perfect crystal:

\[
\begin{align*}
\frac{2i}{k} \frac{\partial E_0'}{\partial x} + \chi_k E_0' &= 0, \\
\frac{2i}{k} \frac{\partial E_h'}{\partial x} + \chi_k E_0' &= 0.
\end{align*}
\]  
(5)

From the continuity condition of the incident and transmitted waves on the entrance surface \( z = 0 \) (s0 = −s0), it follows that

\[
E_0(x, 0) \exp(iK_0x) = E'(x) \exp(iK_x), \quad E_h(x, 0) = 0,
\]  
(6)

where \( E' \) is the amplitude and \( K \) is the wavevector of the incident wave \((K_0 = k \sin \theta, \ K = k \cos \theta)\). Thus, the amplitudes of the transmitted and diffracted waves on the entrance surface are

\[
\begin{align*}
E_0'(x, 0) &= E'(x) \exp \left[ i k \cos \theta \left( \Delta \theta - \frac{T \tan \theta}{2R} \right) x - \frac{ikx^2 \cos \theta}{2R} \right], \\
E_h'(x, 0) &= 0,
\end{align*}
\]  
(7)

where \( \Delta \theta = \theta - \theta' \) is the deviation of the incident wave from the exact Bragg orientation of the perfect crystal. If \( \Delta \theta = T \tan \theta/(2R) \) the incident wave falls under the exact Bragg condition corresponding to the curved crystal.

The solution of (5) for the diffracted wave amplitude with the boundary conditions (7) can be written in the form (Takagi, 1969; Balyan, 2010)

\[
E_h^+ = \int_{-\infty}^{+\infty} G(x-x', z) E'(x') \exp \left[ i k \cos \theta \left( \Delta \theta - \frac{T \tan \theta}{2R} \right) x - \frac{ikx^2 \cos \theta}{2R} \right] dx',
\]  
(8)

where \( \Delta \theta_i = \Delta \theta - T \tan \theta/(2R) \) and the Green’s function is

\[
G(x, z) = \frac{ik\lambda_0 C}{4 \sin \theta} J_0(\pi \cos \theta \sqrt{z^2 \tan^2 \theta - x^2 / \Lambda}) H(z \tan \theta - |x|).
\]  
(9)

Here \( C = 1 \) for \( \sigma \) polarization and \( C = \cos 2\theta \) for \( \pi \) polarization, \( \Lambda = \lambda \cos \theta/(C \sqrt{\lambda_0 \lambda_k}) \). \( H(x) \) is the step function, \( H(x) = 1 \), if \( x > 0 \), \( H(x) = 0 \), if \( x < 0 \), \( J_0 \) is the zero-order Bessel function. From (8) and (9), the diffracted wave amplitude \( E_{h, d}(x, z) = E_h^+(x, z) \exp[ik\lambda_0 z/(2 \cos \theta) - if_i(s_0)] \) for an incident spherical wave is

\[
E_{h, d} = A \exp \left[ \frac{ik\lambda_0 z}{2 \cos \theta} - if_i(s_0) \right] \left[ \int G(x-x', z) \times \exp \left( ik \cos \theta \Delta \theta_i x' - \frac{ikx^2 \cos \theta}{2L_s} - \frac{ikx^2 \cos \theta}{2R} \right) dx' \right] + \int z G(x-x', z) \exp \left( ik \cos \theta \Delta \theta_i x' + \frac{ikx^2 \cos \theta}{2L_s} - \frac{ikx^2 \cos \theta}{2R} \right) dx'.
\]  
(10)

Here \( L_s \) is the source-to-crystal distance, \( A = E_0/L_s \) is the constant amplitude. Also it must be the case that \( z \leq T \).

The area where the two slits contribute with their whole widths is shown in Fig. 1 as the area AB. For the observation points of the area AB the integrations are performed with respect to the widths of the first and second slits. The boundaries of the slits are \((-c - a, -c + a)\) and \((c - a, c + a)\).

### 3. Infinitely narrow slits

For infinitely narrow slits, the integrands in (10) can be taken out under the integral signs at the points \( x' = \pm c \). Thus, in this case

\[
E_{h, d} = 2aA \exp \left[ \frac{ik\lambda_0 z}{2 \cos \theta} - if_i(s_0) + \frac{ikc^2 \cos^2 \theta}{2L_s} - \frac{ikc^2 \cos \theta}{2R} \right] \times \left[ G(x+c, z) \exp(-ik \cos \theta \Delta \theta_i c) + G(x-c, z) \exp(ik \cos \theta \Delta \theta_i c) \right].
\]  
(11)

This expression has an essential difference in comparison with the corresponding formula in the case of a perfect crystal (Balyan, 2010). Instead of \( \Delta \theta \) here we have \( \Delta \theta_i \). Thus, as in the case of a perfect crystal, we can conclude that in the middle of the region AB Young’s double-slit dynamical diffraction fringes are formed. The maxima of the weakly and strongly absorbing modes are defined from the relation

\[
\frac{\pi xc}{\Lambda_t \tan \theta} \pm kc \cos \theta \Delta \theta_i = n\pi, \ n = 0, \pm 1, \pm 2, \ldots
\]  
(12)

Here \( \Lambda_t = \text{Re} \Lambda \) and the sign ‘+’ corresponds to the weakly absorbing mode. As in the case of a perfect crystal, the period of the fringes is

\[
D = \frac{\Lambda_t \tan \theta^2}{c}.
\]  
(13)

The period is different for different polarizations. Consider the case when the crystal thickness is sufficiently large that we need only to take into account the weakly absorbing mode of \( \sigma \) polarization. Then in (12) only the ‘+’ sign can be taken.

From (12) one can make an essential conclusion that the position of the fringes essentially depends on the thickness and on the curvature radius. The fringes are shifted from the centre \( x = 0 \) by

\[
\Delta x = -\frac{k\Lambda_t \tan \theta \cos \theta \Delta \theta - T \tan \theta/(2R)}{\pi}.
\]  
(14)
Knowing the thickness and the deviation from the exact Bragg orientation $\Delta \theta$ and by measuring the shift, one can determine the curvature radius $R$. In practice, one can attempt to obtain the fringes which are not shifted from the centre $[\Delta \theta = T \tan \theta/(2R)]$ and thus knowing the corresponding $\Delta \theta$ and $T$ one can determine the curvature radius $R$.

4. Slits with finite size

However, the slits have finite size $2a$ and it is necessary to discuss the dependence of the interference fringes on the size of the slits. Comparing (10) with the corresponding formula for a perfect crystal (Balyan, 2010), we see that the difference arises not only from $\Delta \theta$ and $\Delta \theta_1$ but also from the fact that under the integral sign in (10) we have the term $\exp[-ikx^2 \cos \theta/(2R)]$. According to (10) for the amplitudes of waves emerging from slit 1 and slit 2 we have

$$E_{h \text{slit}} = A \exp \left[ \frac{ikx_0z}{2 \cos \theta} - ikf_1(x_0) \right]$$

$$\times \int_{-a}^{a} G(x \pm c - x', z) \exp[ik(x' \mp c)^2 \cos^2 \theta/(2L_s)] dx' - ik(x' \mp c)^2 \cos \theta/(2R) \exp[ik \cos \theta(x' \mp c)\Delta \theta_1] dx'.$$

The upper sign corresponds to slit 1 and the lower sign to slit 2. Recall that the intensity in the overlapping region AB is $I = |E_{h \text{slit}}|^2 = |E_{h \text{slit1}} + E_{h \text{slit2}}|^2$. According to the analyses given by Balyan (2010), for sufficiently thick crystals the amplitudes of waves emerging from slits 1 and 2 are proportional to

$$E_{h \text{slit1,2}} \sim \sin \gamma_{1,2}/\gamma_{1,2},$$

where $\gamma_{1,2} = ka \cos \theta \Delta \theta_1 \mp (c \cos \theta/L_s) \pm \pi(x \pm c)/(kA_z \tan^2 \theta \cos \theta)$. From (16) it follows that the coordinates of the maxima of the intensities from each slit can be obtained from the conditions $\gamma_{1,2} = 0$ and are $x_{\text{max1,2}} = \mp c - kA_z \tan^2 \theta \cos \theta \Delta \theta_1 / \pi \pm \pm kA_z \tan^2 \theta \cos^2 \theta [1/L_s - 1/(R \cos \theta)] / \pi$. The distance $\Delta x_{\text{max}} = x_2 - x_1$ between these two maxima will be

$$\Delta x_{\text{max}} = 2c[1 - kA_z \tan^2 \theta \cos^2 \theta [1/L_s - 1/(R \cos \theta)] / \pi].$$

The overlapping region of fields emerging from slits 1 and 2 depends on the source-to-crystal distance and on the curvature radius. As is seen from (17), the distance between maxima is $2c$ for perfect crystals and for sources placed at a large distance. Note that the distance between the maxima is zero when

$$\frac{1}{L_s} - \frac{1}{R \cos \theta} = \frac{\pi}{kA_z \tan^2 \theta \cos^2 \theta}.$$  

(18)

This is the case of maximal overlapping of the interfering waves. From (16) it follows that the width of the intensity distribution for each of the slits is the region between two zeros of the intensity. It is determined from the conditions $\gamma_{1,2} = \pm \pi$:

$$\Delta x = \frac{2A_z \tan^2 \theta}{a}.$$  

(19)

This quantity does not depend on the curvature radius or on the source-to-crystal distance.

5. Some estimates

To understand when the finite size of the slits does not essentially affect the interference fringes, some estimates are needed. First, one must take

$$c \gg a.$$  

(20)

Near the centre of the region AB, where $|x \pm c - x'| < z \tan \theta$, one can use the asymptotic behaviour of Green’s function. Requiring that Green’s function does not essentially change under the integral sign in (15), the following estimate for the weakly absorbing mode is obtained (Balyan, 2010):

$$c < \frac{A_z \tan^2 \theta}{\pi a}.$$  

(21)

From the requirement that the exponentials under the integral signs in (15) are also not essentially changed, the estimates

$$|\Delta \theta_1| < \frac{1}{k \cos \theta a}$$

(22)

and

$$\left| \frac{1}{L_s} - \frac{1}{R \cos \theta} \right| < \frac{1}{kca \cos^2 \theta}$$

(23)

are obtained. In addition, it will be more convenient to choose $\Delta \theta_1$ so that the displacement of fringes (14) is less than the period of the fringes. This leads to the requirement

$$|\Delta \theta_1| < \frac{\pi}{k \cos \theta}.$$  

(24)

Thus from (22) and (24) one must choose the minimal value of $\pi/(kc \cos \theta)$ and $1/(ka \cos \theta)$. However, in most cases $c/\pi > a$ and in most cases $\pi/(kc \cos \theta) < 1/(ka \cos \theta)$ will be realized.

6. Temporal and spatial coherence requirements

The real waves are not monochromatic and the real sources have finite sizes. Thus, the requirements on the monochromatic mode of the incident wave (temporal coherence) and on the size of the source (spatial coherence) must be given. Since the periods in the curved crystals and in the perfect crystals are the same, the coherence requirements must be the same for both cases. Although these requirements have been thoroughly considered by Balyan (2010), here we briefly discuss the same question for the case of curved crystals.

6.1. Temporal coherence requirement

Let the source emit waves with wavelengths in the range $\lambda \in (\lambda_m - \Delta \lambda_m, \lambda_m + \Delta \lambda_m)$ with $|\Delta \lambda/\lambda| \ll 1$, where $\lambda_m$ corresponds to the component with maximal intensity. From the Bragg formula one obtains
\[ \Delta \theta(\lambda) = -\frac{\Delta \lambda}{\lambda} \tan \theta, \]  

where, without loss of generality, \( \Delta \theta(\lambda_m) = 0 \) is set. The shift of the fringes given by (14) and determined only with non-monochromaticity is

\[ \Delta \lambda = \frac{k \Delta \lambda_z \tan^3 \theta \cos \theta \Delta \phi}{\pi}. \]  

This shift is the same as in the perfect crystal and, as in Balyan (2010), one can conclude that the requirement of temporal coherence is

\[ \left| \frac{\Delta \lambda}{\lambda} \right| < \frac{\pi}{2 kc \sin \theta}. \]  

6.2. Spatial coherence requirement

Consider a source with a size \( l \). In the diffraction plane, the source is a line perpendicular to the propagation direction of the incident beam. The coordinates of the point sources \( \xi_i \) along the source vary from \(-l/2\) to \(l/2\). The deviation from Bragg’s exact angle for a point source is

\[ \Delta \theta(\lambda, \xi_i) = -\frac{\Delta \lambda}{\lambda} \tan \theta - \frac{\xi_i}{L_a}. \]  

For a fixed \( \lambda \) the incoherent point sources with the coordinates \( \xi_i = -l/2 \) and \( \xi_i = l/2 \) form interference fringes shifted with respect to each other by \( k c l D \cos \theta/(\pi L_a) \). The interference fringes disappear when \( k c l D \cos \theta/(\pi L_a) = D \). Therefore, the condition for observation of the interference fringes with a high contrast is

\[ l < \frac{\pi L_a}{k c \cos \theta}. \]  

7. Example

Let us consider an example of double-slit dynamical diffraction in a curved crystal. We will consider the plane wave case. This case can be realized using one or two asymmetrical Bragg reflections and preparing a collimated and monochromatic beam. This can be done using synchrotron and laboratory sources of X-rays. For instance, let us take the reflection \( \text{Si}(220) \), \( \lambda = 0.708 \text{ Å}, \ \theta = 10.6^\circ, \ L_{\sigma} = 36.6 \text{ μm}, \ \) incident plane wave of \( \sigma \) polarization, \( 2a = 10 \text{ μm}, \ z = T = 5 \text{ mm} \). The necessary silicon data \( \chi_{00} = -3.162 \times 10^{-6}, \ \chi_{0} = 0.165 \times 10^{-7}, \ \chi_{0z} = -1.901 \times 10^{-6}, \ \chi_{0i} = 0.159 \times 10^{-7} \) are taken from Pinsker (1982). The absorption coefficient \( \mu = k \chi_{00} \) and for the chosen thickness \( \mu z = 7.3 \). Therefore, the weakly absorbing mode mainly contributes to the intensity. If we take \( 2c = 80 \text{ μm} \), the period (13) of interference fringes will be \( D = 161 \text{ μm} \). The requirement (20) for temporal coherence is \( |\Delta \lambda/\lambda| < 2.4 \times 10^{-6} \). From the spatial coherence requirement (29) for a source with the size \( 30 \text{ μm} \) we have \( L_a > 33 \text{ m} \). The requirements (20) and (21) are fulfilled. The requirement (22) in this case is satisfied if \( |\Delta \theta| < 2.3 \times 10^{-6} \) and the condition (24) is fulfilled if \( |\Delta \theta| < 9 \times 10^{-7} \). From (23), the condition \( |R| > 17.4 \text{ m} \) for the radius is obtained. The width of the overlapping region \( AB \) is \( 2 \times 893 \text{ μm} \approx 1.8 \text{ mm} \). The number of fringes is approximately \( 11 \). However, if we decrease \( 2c \), the radius minimal value starting from which the fringes can be formed will decrease, but the period will increase and the number of fringes will decrease as well. If we take \( 2c = 40 \text{ μm} \) the condition (23) will be fulfilled for \( |R| > 8.7 \text{ m} \). The period is \( 322 \text{ mm} \) and the width of the overlapping region \( AB \) will be approximately \( 2 \times 913 \text{ μm} \approx 1.8 \text{ mm} \) and thus the number of fringes will be approximately \( 6 \). The requirements (20) and (21) are fulfilled in this case. The condition (24) results in the values \( |\Delta \theta| < 1.8 \times 10^{-6} \). The radius, given by (18), for which the intensity distributions from two slits are fully overlapped for both cases of \( c \) is \( R \approx -179 \text{ m} \).

We will present here some results of numerically calculated intensity distributions \( I = |E_{d2}|^2 = |E_{d2} + E_{d1}|^2 \), using the exact formula (10). These results will be compared with the analytical-based predictions given in Sections 3–5. The intensities will be given in the units \( |A|^2 \).

First, let us consider the case when the distance between the slit centres is \( 2c = 80 \text{ μm} \). In Fig. 2(a), the intensity distributions of waves emerging from slit 1 and slit 2 are separately shown. According to the requirements (23) \( |R| > 17.4 \text{ m} \) and the curvature radius is set \( R = -20 \text{ m} \). The curvatures with

Figure 2

\( 2c = 80 \text{ μm}, \ R = -20 \text{ m}, \ \Delta \theta_1 = 0 \). Numerically calculated intensity distributions: (a) separate slits; (b) Young’s fringes.
negative values of the radius correspond to the bending type shown in Fig. 1. The deviation \( \Delta \theta_i = 0 \) is taken, i.e. \( \Delta \theta = T \tan \theta/(2R) = -2 \times 10^{-5} \). The corresponding dynamical diffraction Young’s fringes are shown in Fig. 2(b) where one can see the agreement of the obtained results of numerical calculations with the approximate analytical predictions. The numerically calculated period in this figure is \( D = 158 \mu m \) against the analytically obtained approximate value \( D = 161 \mu m \), given by the formula (13). Since \( \Delta \theta_i = 0 \), the Young’s fringes are not shifted from the line \( x = 0 \) which is analytically predicted by the formula (14). The intensity of waves emerging from the separate slits and the corresponding Young’s fringes for the case \( \Delta \theta_i = 8 \times 10^{-7} \) (\( \Delta \theta = -2 \times 10^{-5} + 8 \times 10^{-7} \)) are shown in Figs. 3(a) and 3(b).

According to (22), to obtain the fringes one can choose any value \( |\Delta \theta_i| < 2.3 \times 10^{-6} \). But according to (24) the shift will be less than the period if \( |\Delta \theta_i| < 9 \times 10^{-7} \). That is why we choose the value \( \Delta \theta_i = 8 \times 10^{-7} \). It can be seen from Fig. 3(a) that, due to non-zero \( \Delta \theta_i \), the intensities from individual slits, in accordance with (16), are not equal. The corresponding Young’s fringes, as seen from Fig. 3(b), are shifted from the line \( x = 0 \) by \( \Delta x = -137 \mu m \). The analytical value, predicted by formula (14), is \( \Delta x = -143 \mu m \).

To obtain interference fringes for a lower curvature radius of the crystal, consider the case \( 2c = 40 \mu m \). In this case, the fringes can be obtained for the values \( |R| > 8.7 \mu m \). This is predicted by the formula (23). The numerically obtained Young’s fringes for \( R = 10 \mu m \) and for the case \( \Delta \theta_i = 0 \) \( (\Delta \theta = 4.7 \times 10^{-5}) \) are shown in Fig. 4(a). The calculated period of the fringes in this figure is \( D = 292 \mu m \). The analytical value, according to formula (13), is \( D = 322 \mu m \). In Fig. 4(b) the numerically calculated interference fringes corresponding to \( \Delta \theta_i = -10^{-6} \) \( (\Delta \theta = 4.7 \times 10^{-5} - 10^{-6}) \) are shown. We choose this value since, according to formula (24), the shift will be less than the period if \( |\Delta \theta_i| < 1.8 \times 10^{-6} \). The interference fringes in this figure are shifted from the line \( x = 0 \) by \( \Delta x = 172 \mu m \). The predicted theoretical value, given by formula (14), is equal to \( \Delta x = 179 \mu m \).

8. Conclusion

In this paper, the double-slit dynamical diffraction of X-rays in curved crystals in the symmetrical Laue case is theoretically investigated. On the exit surface of the crystal, the interference fringes similar to Young’s fringes are formed and an analytical formula for the period of the fringes is obtained. The period does not depend on the curvature radius and is polarization sensitive. The position of the fringes in the beam
depends on the deviation from exact Bragg orientation for a perfect crystal, on the thickness of the crystal and on the curvature radius. Knowing the deviation from the Bragg exact orientation, the thickness of the crystal and by measuring the shift of the fringes from the centre of the beam, the curvature radius can be determined.

The double-slit dynamical diffraction in deformed and imperfect crystals is a sensitive tool to determine the characteristics of deformations using the diffraction contrast formed by two slits. This is an additional method to the one-slit contrast method for deformed crystals. The double-slit interference pattern in deformed crystals can be considered as a hologram. In principle, this can be used for solving the inverse problem to determine the deformations.

References


